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Determination of The Best Box-Jenkins' Seasonal Model to Inflows of Reservoir of Bekhme Dam

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Abstract. Prediction of water inflow to a reservoir is of great interest in the policy of the reservoir operation throughout the year. When significant amounts of inflow series entering to the reservoir are nondeterministic events, the utilization of stochastic models to check the reliability of the recorded data and forecast the future events become preferable. One of the most powerful and widely used methodology for forecasting time series is the class of models called the Box-Jenkins models. In this study, time series analysis was applied to records of 69 monthly mean inflows to Bekhme reservoir, in the northern part of Iraq, for the water year period from 1933 to 2006. Nine multiplicative seasonal models were fitted to this series; these were the seasonal autoregressive integrated (SARI) (1, 1, 0) × (1, 1, 0)12, (2, 1, 0) × (1, 1, 0)12, and (1, 1, 0) × (2, 1, 0)12 models, the seasonal autoregressive integrated moving average (SARIMA) (1, 1, 1) × (1, 1, 1)12, (2, 1, 2) × (1, 1, 1)12, and (1, 1, 1) × (2, 1, 2)12 models, and the seasonal integrated moving average (SIMA) (0, 1, 1) × (0, 1, 1)12, (0, 1, 2) × (0, 1, 1)12, and (0, 1, 1) × (0, 1, 2)12 models. The unconditional sum of squares method was used to estimate the parameters of the models and to compute the sum of squared errors for each one. It was found that the best model which corresponded to the minimum sum of squared errors was the SIMA (0, 1, 1) × (0, 1, 1)12 model. The estimated moving average parameters of this model were 0.378 and 0.953 for both θ and Θ respectively. The adequacy of this model was checked by plotting the normalized cumulative periodogram which does not indicate nonrandomness of the residuals. Forecasts of monthly inflow for the period from October, 2002, to September, 2006 were graphically compared with observed inflow for the same period and since agreement was very precise, adequacy of the selected model was confirmed.

Keywords: Box-Jenkins; Bekhme dam; seasonal model; ARIMA; SARIMA

1. Introduction

A time series is a collection of observations made sequentially in time. If the set is continuous, the time series is said to be continuous. If the set is discrete, the time series is said to be discrete. If future values of a time series are exactly determined by some mathematical function such the time series is said to be deterministic. If the future values can be described only in terms of probability distribution, the time series is said to be nondeterministic or simply a statistical time series (Box and Jenkins, 1976).

The time series may consist of four components depending on the type of variable and the averaging time interval. These components may exist in monthly time series, which may be formulated by (Al-Ta'ee, 2009):

$$X_t = J_t + T_t + P_t + W_t \quad (1)$$

where X_t is the time series observations at time t (= 1, 2, 3, …, N), J_t is the jump component, T_t is the trend component, P_t is the periodic or seasonal component, W_t is the stochastic component included dependent and independent part, and N is the no. of observations.

When the components are nonlinearly related, the relationship (Equation 1) can often be made linear by taking logarithms (Jayawardena and Lai, 1989). The Jump, trend, and periodic components represent the deterministic part of the process while the stochastic component represents the nondeterministic part. Therefore, the first three components should be detected and identified by suitable formulations and decomposed from the stochastic component (Al-Ta'ee, 2009). The stochastic component contains a dependent and an independent part. The dependent part may be represented by one of the widely linear time series models such as the Box-Jenkins models. The independent part can only be described by some probability distribution functions.

In the 1960s, Box-Jenkins developed a general model for the most time series, without needing to assume an initially fixed pattern and to limit a specific kind of pattern, which was known as an Autoregressive Integrated Moving Average model and denoted it as ARIMA (p, d, q) model, where p and q is the order of the autoregressive and moving average models, respectively, and d is the degree of differencing. The ARIMA (p, d, q) model may be for singlesite, multisite, univariate, or multivariate stochastic process. For a stationary series, the ARIMA (p, d, q) model can be shortened as an Autoregressive Moving Average (ARMA (p, q)) model (Sarby et al., 2007). Box and Jenkins (1976) generalized the ARIMA (p, d, q) model to deal with seasonality in the time series. They defined a general multiplicative seasonal model that was denoted it as ARIMA (p, d, q) × ARIMA (P, D, Q)$_s$, where p, q , d and P, D, Q refer to the order of nonseasonality and seasonality (for a specified lag need to be estimated) in the time series, respectively (Sabry et. al., 2007). The general equation of this model is as follows:

$$\phi_p(\beta)\Phi_P\left(\beta^S\right) W_t = \theta_q(\beta)\,\Theta_Q\left(\beta^S\right) a_t \quad (2)$$

where ϕ_p is the parameter of the autoregressive term of order p, Φ_P is the parameter of the seasonal autoregressive term of order P, θ_q is the parameter of the moving average term of order q, Θ_Q is the parameter of the moving average term of order Q, β is the backward shift operator, S is the seasonal or periodic cycle such as 6, 12, 24 months, etc. , and a_t is the purely random process (white noise process or shock process), independent part of the stochastic component, with mean zero and variance σ_a^2.
According to Box and Jenkins (1976) the values of ϕ_p, Φ_P, θ_q, and Θ_Q are restricted between -1 and +1 (Mahpol, 2005). The values of W_t are derived by differencing the normalized series to remove both the trend and seasonal component as shown in the following equation:

$$W_t = \nabla^d \nabla_s^D Z_t \quad (3)$$

where ∇^d is the backward difference of order d, d is the maximum number of differencing (for the trend removal) to make the time series stationary in the mean, ∇_S^D is the seasonal backward difference of order D and season S, D is the degree of seasonal differencing (for the seasonality removal), and Z_t is the normalized time series that is derived from the original time series (X_t) by using natural log, square root, Box-Cox transformation, etc.; the more suitable transformation is natural log transformation. Hence,

$$Z_t = \ln(X_t) \quad (4)$$

Now, Equation 2 can be rewritten as follows:

$$\varphi_p(\beta)\Phi_P(\beta^s)\nabla^d\nabla_s^D Z_t = \theta_q(\beta)\Theta_Q(\beta^s)a_t \quad (5)$$

or

$$(1-\sum_{j=1}^{p}\varphi_j\beta^j)(1-\sum_{j=1}^{P}\Phi_j\beta^j)\nabla^d\nabla_s^D Z_t = (1-\sum_{j=1}^{q}\theta_j\beta^j)(1-\sum_{j=1}^{Q}\Theta_j\beta^{sj})a_t \quad (6)$$

This model, i.e., Equation 5 or 6, is said to be of order (p, d, q) × (P, D, Q)$_s$
Now, if p = P = 1, d = D = 1, q = Q = 1, and S = 12, Equation 6 becomes:

$$(1-\varphi\beta)(1-\Theta\beta^{12})\nabla\nabla_{12}Z_t = (1-\theta\beta)(1-\Theta\beta^{12})a_t \quad (7)$$

This is a multiplicative seasonal (or SARIMA) (1, 1, 1) × (1, 1, 1)$_{12}$ model.
If p = P = 0, d = D = 1, q = Q = 1, and S = 12, Equation 6 becomes:

$$\nabla\nabla_{12}Z_t = (1-\theta\beta)(1-\Theta\beta^{12})a_t \quad (8)$$

This is a multiplicative seasonal (or SIMA) (0, 1, 1) × (0, 1, 1)$_{12}$.
If p = P = 1, d = D = 1, q = Q = 0, and S = 12, Equation 6 becomes:

$$(1-\varphi\beta)(1-\Theta\beta^{12})\nabla\nabla_{12}Z_t = a_t \quad (9)$$

This is a multiplicative seasonal (or SARI) (1, 1, 0) × (1, 1, 0)$_{12}$.
Thus, a multiplicative seasonal of any order can be derived from Equation 6 after the values of of ϕ_p, Φ_P, θ_q, and Θ_Q are defined.

The main objective of this study is to apply the Box-Jenkins seasonal models to the seasonal time series of monthly inflow to Bekhme reservoir in the north of Iraq. Then, the model that gives the best fit to the records of monthly inflow is used to predict the future values of inflow.

2. Description of The Study Site

The Bekhme dam is located at about 7 km upstream of Bekhme village and approximately 2 km downstream of the confluence with the Ruwandus river which flows from the left bank side as shown in Figure 1.

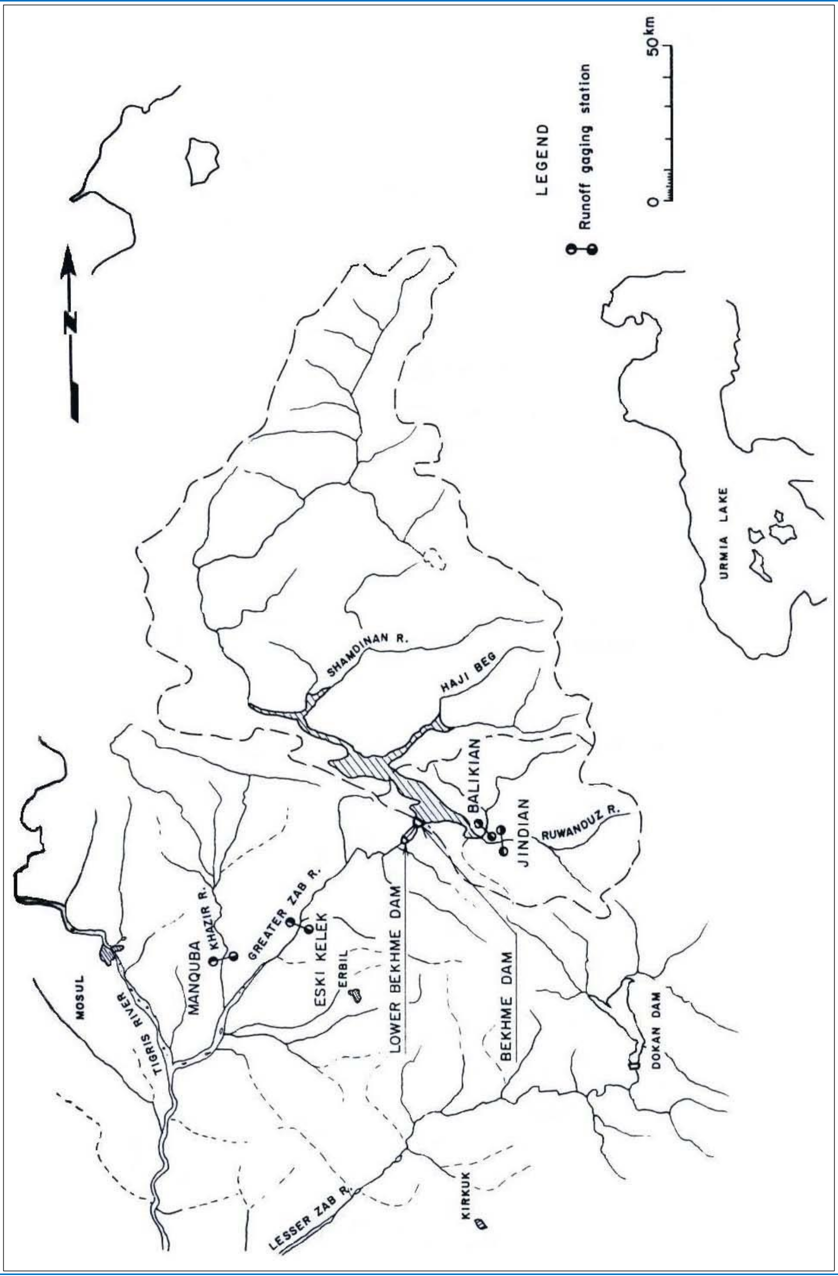

Figure 1: Site Map of Bekhme Reservoir (Iraqi Ministry of Water Resources, 1986)

3. Fitting Box-Jenkins' Seasonal Models

The general method of forecasting does not assume any particular pattern for the historical data of the series. It uses an iterative approach of identifying a possible useful model from a general class of models. Then, the chosen model is checked against the historical data to see if it accurately describes the series. The model is appropriate if the residuals between the forecasting and the historical data points are small (close to zero), randomly distributed, and independent. If the specified model is not satisfactory, the process is repeated until a satisfactory model is found (Chatfield, 1989).

The monthly inflow of Bekhme reservoir for the record of 69 water years, from October 1933 to September 2002, were plotted in Figure 2 to show up important features such as trend, seasonality, discontinuities, and outlier. This figure shows little trend, high fluctuation, and seasonal variation for monthly inflow of Bakheme reservoir. The periodicity of the series was checked on 05 water years from 1933/1934 to 1937/1938, as shown in Figure 3. In this figure, a remarkable seasonal pattern of monthly inflow is clearly visible with the highest value occurring always in the spring (April) of each water year, i.e., similarities occurred after each12 months.

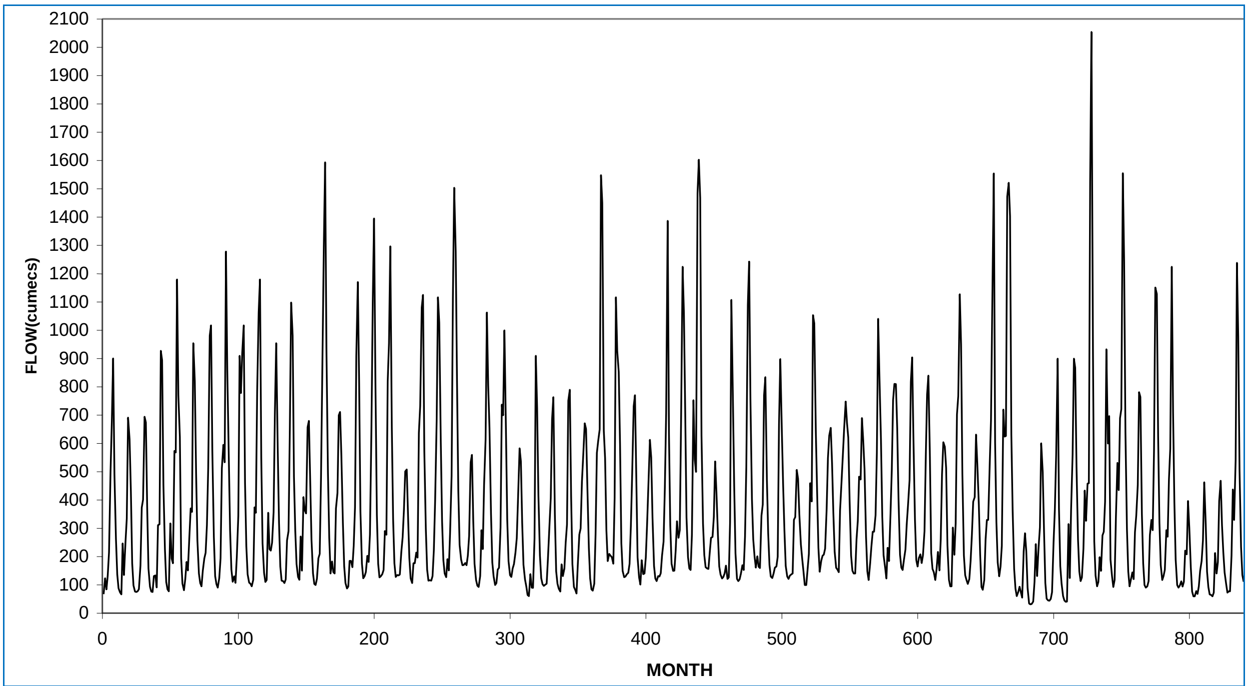

Figure 2: Monthly mean inflow of Bekhme reservoir for 69 water years (from October 1933 to September 2002)

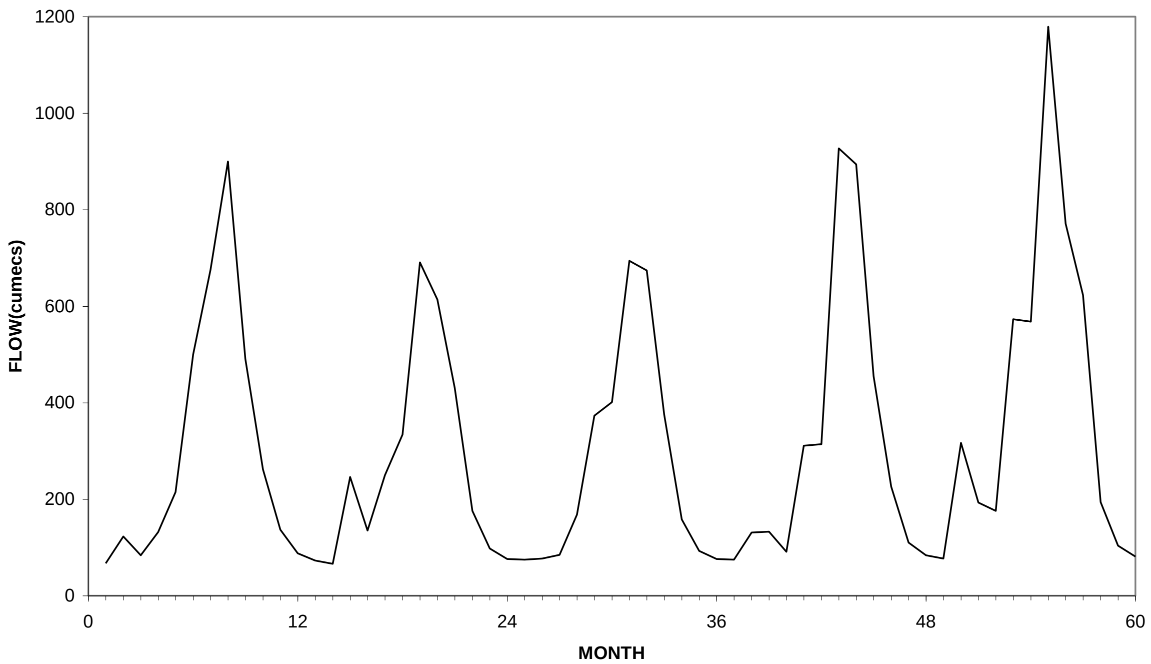

Figure 3: Monthly mean inflow of Bekhme reservoir for 05 water years (from October 1933 to September 1938)

Relating the Box-Jenkins seasonal models to data is carried out by three iterative stages. These stages are identification of model, estimation of model parameters, and diagnostic checking of model (Young, 1974).

3.1 Identification of model

Model identification means use of data, and any information on how the series was generated, to suggest subclasses from the general Box-Jenkins seasonal model (Equation 2) for further examination. In other word, identification provides comprehensive clues about the choice of the orders of p, d, q, P, D, and Q. However, in practice, the degrees of differencing (d and D) are assumed to be one and the autocorrelation and partial autocorrelation functions are plotted to estimate the orders of p, q, P, and Q.

The procedure of identification of the Box-Jenkins seasonal models are summarized as follows (Box and Jenkins, 1976):

1. Transforming data by using natural log transformation.
2. Removing trend component by trying the first order differencing.
3. Removing the seasonal variation by trying the first order seasonal differencing.
4. Model identification by plotting autocorrelation function (ACF) and partial autocorrelation function (PACF) of monthly observations.

Figures 4 and 5 show the estimated autocorrelation and autocorrelation functions of the monthl y inflow of Bekhme reservoir which alternate in sign and tend to damp out with increasing lags. This beh avior is similar to that associated with autoregressive process (Chatfield, 1989). Therefore, the SARI (1, 1 , 0) × $(1, 1, 0)_{12}$, (2, 1, 0) × $(1, 1, 0)_{12}$, and (1, 1, 0) × $(2, 1, 0)_{12}$ models were applied to the inflow of Bekh me reservoir. However, the estimated values of the lag 11 correlation coefficient, r_{11} (= 0.148), is very clo se to the estimated value of the lag 13 correlation coefficient, r_{13}(= 0.152), and to the product of the estim ated values of lag 1 and lag 12 correlation coefficients, r_1r_{12} (= -0.288 × - 0.511 = 0.147). Hence, these results appear to indicate a moving average process (Box and Jenkins, 1976) . Therefore, in addition to the seasonal ARI models applied to the series of the inflow, the SIMA (0, 1, 1) × $(0, 1, 1)_{12}$, (0, 1, 2) × $(0, 1, 1)_{12}$, and (0, 1, 1) × $(0, 1, 2)_{12}$ models were also fitted to the inflow series. Th en, for further examinations, the SARIA $(1, 1, 1) \times (1, 1, 1)_{12}$, $(2, 1, 2) \times (1, 1, 1)_{12}$, and $(1, 1, 1) \times (2, 1, 2)_{12}$ models were applied to the series of inflow. Finally, the appropriate model which will give mini mum sum of squared errors (SS).

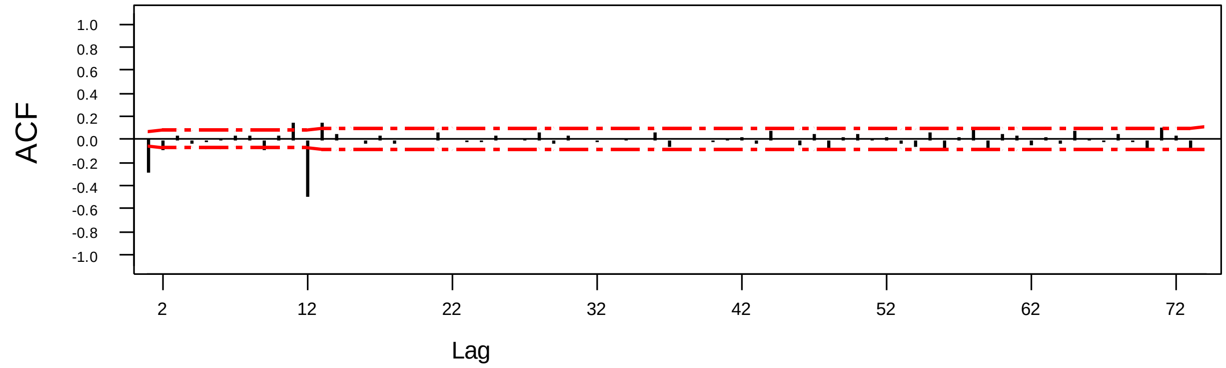

Figure 4: Autocorrelation function of monthly inflow of Bekhme reservoir.

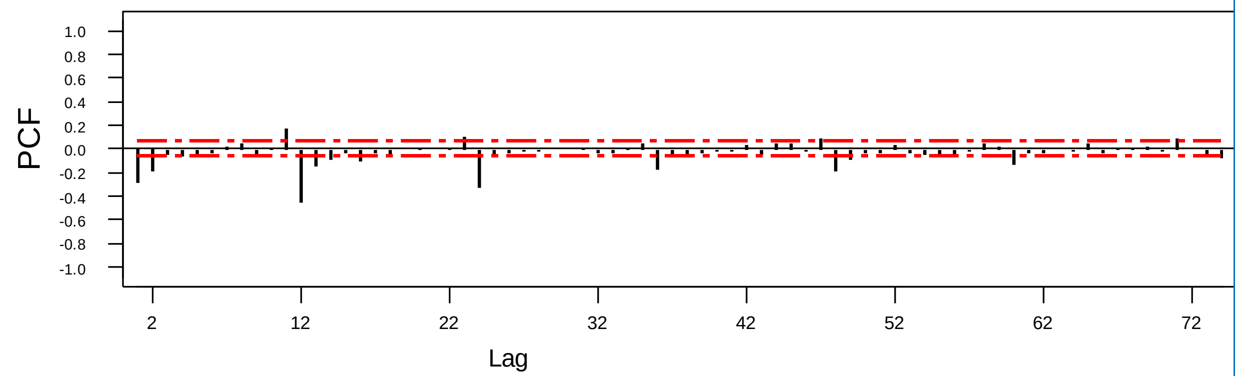

Figure 5: Partial autocorrelation function of monthly inflow of Bekhme reservoir.

3.2 Estimation of model parameters

The unconditional sum of squares method was used to estimate the parameters of the models suggested in Subsection 4.1. The parameters of the model, i.e., ϕ, Φ, θ, and Θ were ranged from -1 to +1 and the sum of squares, SS, was computed. The values of the parameters corresponding to the minimum sum of squares were considered. To illustrate this procedure, the estimation of parameters of the $(0, 1, 1) \times (0, 1, 1)_{12}$ model, i.e., θ and Θ in Equation 8, were given in Table 1 for θ and Θ of 0.378 and 0.953, respectively, corresponding to the minimum sum of squares of 70.675. It was assumed that e_t was 0 for t from 828 to 840, a_t was 0 for t from -13 to -25, and e_t was 0 for t from 0 to -12. The computation in this table started from the bottom of it using either the forward or backward forms of the model which were given below as Equations 10 and 11 respectively:

Table 1: Computation of sum of squares (SS) for the (0, 1, 1) × $(0, 1, 1)_{12}$ model with θ and Θ of 0.378 and 0.953, respectively.

t	X(t)	Z(t) = ln X(t)	W(t)	e(t)	a(t)	a(t)^2
-12	67	4.204693	-0.05312	0	-0.05312	0.002821
-11	123	4.812184	0.323697	0	0.303619	0.092185
-10	84	4.430817	-0.57967	0	-0.4649	0.216136
-9	132	4.882802	0.175361	0	-0.00037	1.39E-07
-8	215	5.370638	0.052108	0	0.051967	0.002701
-7	500	6.214608	0.448716	0	0.46836	0.219361
-6	676	6.516193	-0.24914	0	-0.0721	0.005199
-5	900	6.802395	0.311501	0	0.284246	0.080796
-4	491	6.196444	-0.07895	0	0.028491	0.000812
-3	262	5.568345	0.08075	0	0.09152	0.008376
-2	137	4.919981	-0.07111	0	-0.03652	0.001333
-1	88	4.477337	-0.16903	0	-0.18284	0.033429
0	73	4.290459	-0.13988	0	-0.25961	0.067396
1	66	4.189655	-0.7083	-0.14745	-0.49795	0.247949
			↑	↑	↓	↓
814	104	4.644391	0.006711	-0.25705	0.329498	0.108569
815	72	4.276666	-0.33695	-0.13148	-0.01783	0.000318
816	77	4.343805	0.148485	0.10999	0.017169	0.000295
817	77	4.343805	-0.19913	-0.10184	-0.28362	0.080439
818	203	5.313206	-0.11052	0.257382	0.493696	0.243736
819	437	6.079933	1.181671	0.973284	0.835669	0.698342
820	329	5.796058	-0.53519	-0.55129	-0.33482	0.112105
821	540	6.291569	-0.29295	-0.04259	-0.03152	0.000994
822	1238	7.121252	0.662629	0.662319	0.296769	0.088072
823	977	6.884487	0.17191	-0.00082	0.01205	0.000145
824	504	6.222576	-0.2974	-0.45696	-0.12739	0.016228
825	234	5.455321	-0.36179	-0.42212	-0.1187	0.014091
826	135	4.905275	-0.22462	-0.15962	-0.06964	0.00485
827	111	4.70953	0.17198	0.17198	0.009963	9.93E-05
					SS=	70.675

$$[e_t] = [W_t] + \theta[e_{e+1}] + \Theta[e_{t+12}] - \theta\Theta[e_{t+13}] \quad (10)$$

$$[a_t] = [W_t] + \theta[a_{t-1}] + \Theta[a_{t-12}] - \theta\Theta[a_{t-13}] \quad (11)$$

The results of application of the unconditional sum of squares method to estimate the parameters for the other eight models were shown in Table 2.

Table 2: Estimated values of parameters of eight sessional models corresponding to the minimum sum of squares

Model	Parameters	Sum of squares (SS)
$(1, 1, 0) \times (1, 1, 0)_{12}$	ϕ = -0.285, Φ = -0.510	100.601
$(2, 1, 0) \times (1, 1, 0)_{12}$	ϕ_1 = -0.331, ϕ_2 = -0.174, Φ = -0.511	97.791
$(1, 1, 0) \times (2, 1, 0)_{12}$	ϕ = -0.421, Φ_1 = -0.152, Φ_2 = -0.572	95.233
$(0, 1, 2) \times (0, 1, 1)_{12}$	θ_1 = 0.294, θ_2 = 0.452, Θ = 0.765	80.176
$(0, 1, 1) \times (0, 1, 2)_{12}$	θ = 0.227, Θ_1 = 0.784, Θ_2 = 0.985	75.291
$(1, 1, 1) \times (1, 1, 1)_{12}$	ϕ = -0.358, Φ = -0.694, θ = 0.198, Θ = 0.964	112.957
$(2, 1, 2) \times (1, 1, 1)_{12}$	ϕ_1 = -0.521, ϕ_2 = -0.196, Φ_1 = -0.591, Φ_2 = -0.605, θ = 0.837, Θ = 0.864	93.283
$(1, 1, 1) \times (2, 1, 2)_{12}$	ϕ = 0.118, Φ = 0.328, θ_1 = 0.387, θ_2 = 0.482, Θ_1 = 0.994, Θ_2 = 0.895	118.556

Hence, according to these results it was found that the $(0, 1, 1) \times (0, 1, 1)_{12}$ model agreed with the historical series better than the other eight models. On the other hand, it was concluded that the findings of the current study completely agreed with those obtained by Ali (2009), which showed that the same model with the same values of parameters obtained in this study. However, a diagnostic test should be performed before a final decision on the most appropriate model is made.

3.3 Model diagnostic checking

The periodogram, $I(f_i)$, of a time series of residuals (a_t), t= 1, 2, ……., n, is defined as (Chatfield and Prathero, 1973):

$$I(f_i) = \frac{2}{n}[(\sum_{t=1}^{n} a_t Cos2\pi f_i t)^2 + (\sum_{t=1}^{n} a_t Sin2\pi f_i t)^2] \quad 12$$

where f_i = (i/n) is the frequency and $i = 1, 2, \ldots\ldots, (\frac{n-1}{2})$.

The normalized cumulative periodogram which is an effective means for detection of periodic nonrandomness is given by Barnlett (Chatfield, 1989) as:

$$C(f_i) = \frac{\sum_{i=1}^{j} I(f_i)}{ns^2} \quad 13$$

where s^2 is an estimate of the variance σ_a^2.

If the estimated values of the residuals were purly random, i.e., white noise, deviation of the cumulative periodogram from that expected is assessed from plot of a C(f_i) versus fi. Thus, limit lines are placed about the theoretical line corresponding to white noise such that if the residual series were white noise, the C(f_i) would deviate from the straight line sufficiently to cross these limits only with a stated probability. The limit lines are drawn at distances $\pm K_\xi\sqrt{[(n-1)/2]}$ above and below the theoretical line. For 5% probability limits, i.e., ξ= 0.05, K_ξ is approximately 1.36.

The plot of normalized cumulative periodogram of the residuals from the $(0, 1, 1) \times (0, 1, 1)_{12}$ model fitted to the inflow series with θ and Θ of 0.378 and 0.953, respectively, was shown in Figure 6 which failed to indicate any significant departure from the assumed model.

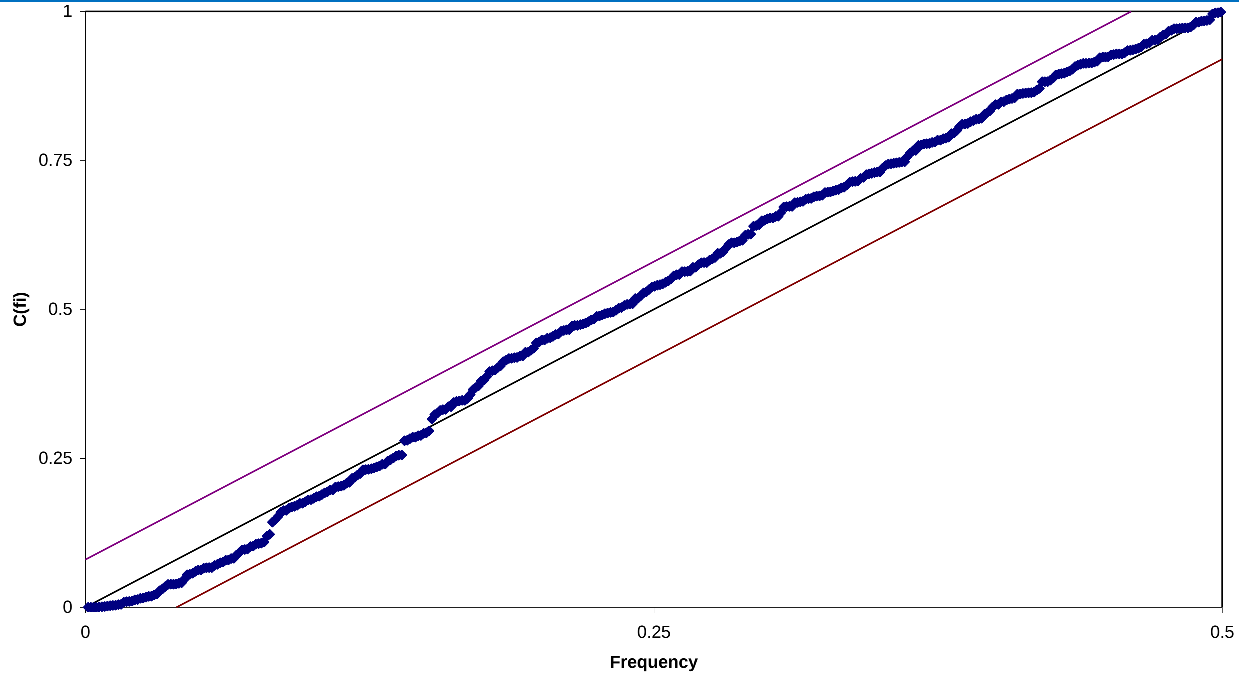

Figure 6: Cumulative Periodogram of residuals from (0, 1, 1) × (0, 1, 1)$_{12}$ model.

3.4 Confidence regions for the model parameters

The (1-ξ) confidence region for ~~the~~ model parameters is bounded by the contour on the sum of squared surface for which (Chatfield and Prathero, 1973):

$$S(\beta) = S\hat{\beta})\{1 + \frac{\chi^2_{\xi}(K)}{n}\} \quad 14$$

where β is model parameter, $\hat{\beta}$ estimated model parameters, $\chi^2_{\zeta}(K)$ is the Chi squared value corresponding to χ^2 proportion of the distribution with K degrees of freedom, and K is number of estimated parameters.

Equation 14 was used to determine the 95 and 99 percent confidence regions for the model parameters as shown in Figure 7. In this figure, the 71.17 represents the boundary of the 95% confidence region while the 71.44 represents the boundary of the 99% confidence region.

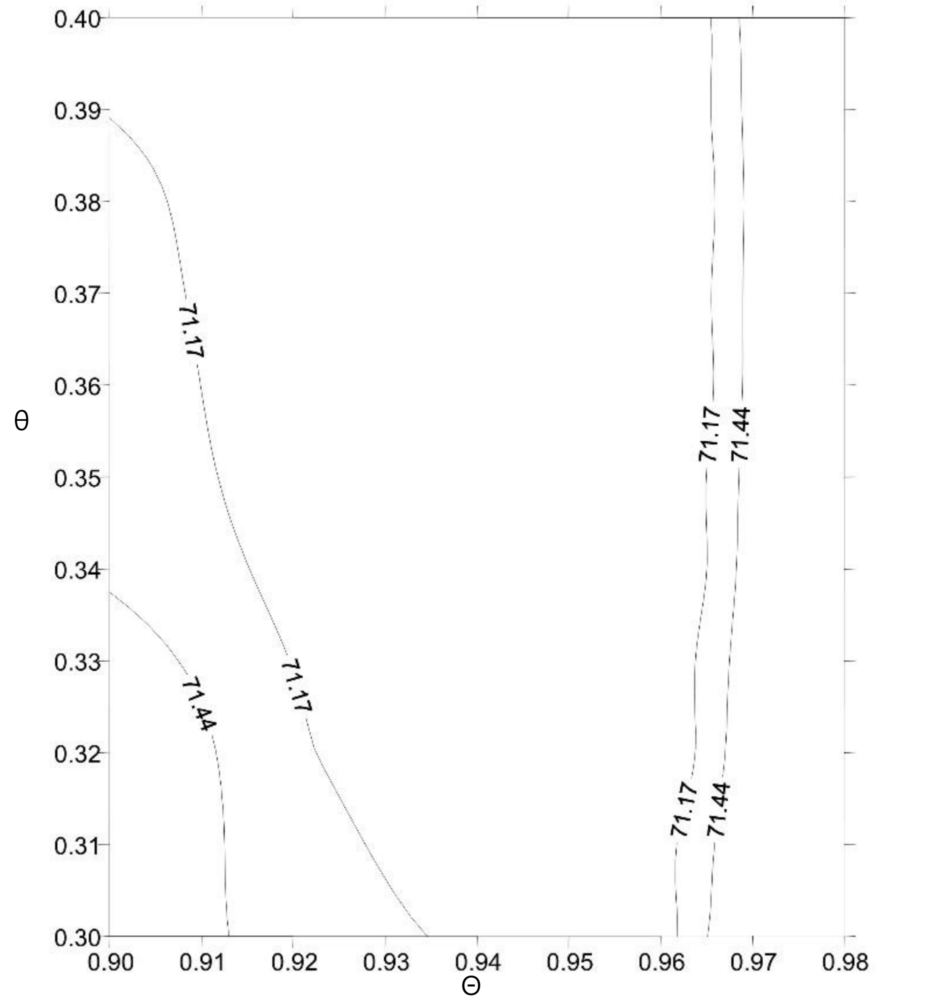

Figure 7: The sum of squares surface showing 95 and 99 percent regions of the parameters of the (0, 1, 1) × (0, 1, 1)$_{12}$ model.

4. Comparison Between Forecasted and Observed Inflows

At time t+ℓ, the (0, 1, 1) × (0, 1, 1)$_{12}$ model which was fitted to the inflow series, i.e., Equation 8 with θ and Θ of 0.378 and 0.953, respectively, gave the following equation:

$$Z_{t+\ell} = Z_{t+\ell-1} + Z_{t+\ell-12} + Z_{t+\ell-13} + a_{t+\ell} - 0.378a_{t+\ell-1} - 0.953a_{t+\ell-12} + 0.378a_{t+\ell-13} \quad 14$$

This equation was used to obtain forecasts with the unknown Z's being replaced by their already forecasted values and unknown a's by zeroes (Box and Jenkins, 1976). Hence, the forecasts were obtained for lead times up to 48 months, all made at the origin of September 2002, as shown in Figure 8. The corresponding observed values were also shown in this figure and since agreement between observed and forecasted values was very good, it was confirmed that the model was adequate.

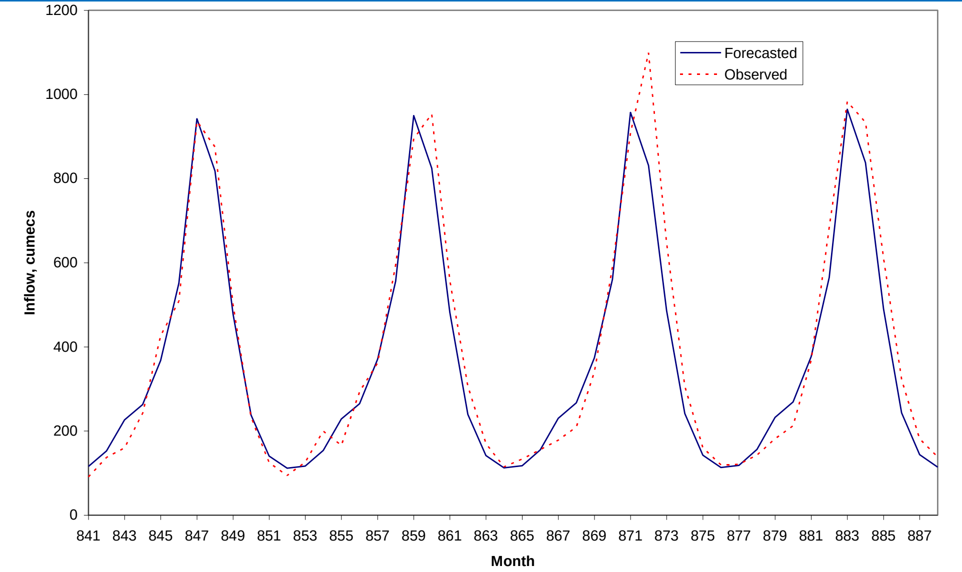

Figure 8: Comparison of forecasted and observed inflows (from October 2022 to September 2006).

6. Conclusions

Based on results in this study, the following conclusions are summarized:

1. The time series of monthly inflow to Bekhme reservoir was a periodic series. The seasonal pattern may be due to the influence of the annual cyclic pattern of the hydrological inputs to the reservoir.
2. Among nine Box-Jenkins seasonal models, it was found that the $(0, 1, 1) \times (0, 1, 1)_{12}$ model was the best fitted one and completely agreed with that obtained by Ali (2009).
3. The diagnostic checking of cumulative periodogram indicated no significant departure of the series from the selected model.
4. The forecasted future inflow to Bekheme reservoir could obtained from the $(0, 1, 1) \times (0, 1, 1)_{12}$ model in high confidence level.

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Conflicts of Interest: The authors declare no conflict of interest.

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Nomenclatures

a_t	purely random process (white noise process or shock process)
ACF	autocorrelation function.
ARIMA	autoregressive integrated moving average.
$C(f_i)$	cumulative periodogram.
d	degree of differencing of ARIMA model.
D	degree of seasonal differencing of SARIMA model.
$I(f_i)$	periodogram.
PACF	partial autocorrelation function.
p, q	orders of ARIMA model.
P, Q	orders of SARIMA model.
s	number of seasons.
SARI	seasonal autoregressive integrated.
SARIMA	seasonal autoregressive integrated moving average.
SIMA	seasonal integrated moving average.
SS	sum of squared errors.
W_t	seasonal component of time series.
X_t	original time series.
Z_t	normalized time series (=ln (X_t)).

Greek symbols

β	backward shift operator.
Ø, Φ	autoregressive parameters.
θ, Θ	moving average parameters.
∇	backward difference operator.
σ_a^2	coefficient of variance.